

Name: \_\_\_\_\_

Score: \_\_\_\_\_

1. (a) Sketch the direction field for the differential equation  $y' = y^2 - 1$ . (2 points)
- (b) Sketch the solution curves passing through  $(0, 0)$  and  $(1, 1)$  respectively. (2 points)
- (c) Can a solution to the differential equation  $y' = y^2 - 1$  tend to  $-1$  as  $x \rightarrow \infty$ ? Explain your answer using the direction field in part a. (2 points)

2. Let  $f(x) = e^x, g(x) = xe^x$ .

- (a) Verify that  $f(x)$  and  $g(x)$  are both solutions to the differential equation  $y'' - 2y' + y = 0$ . (4 points)
- (b) \* Show that for any real numbers  $a, b$ ,  $af(x) + bg(x)$  is also a solution to the differential equation  $y'' - 2y' + y = 0$ . (Bonus 1 point)